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A Usage Based Framework for Choosing Appropriate Rotation Schemes in Factor Analysis

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Abstract

This paper provides users of factor analysis with an application oriented framework for choosing an appropriate rotation.

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Introduction

Factor analysis plays an important role in marketing research. It is widely used both by academicians and practitioners. While the basic models used in factor analysis are well grounded in mathematical theories of linear algebra and statistics, considerable maturity and judgment is required in interpreting the results of a factor analysis "solution." For example, Dielman, Cattell, and Wagner (1972) introduce their paper saying, "It is not unusual for the type in factor analysis to consider rotation as just one of half-a-dozen technical steps in the whole analysis. No greater mistake could be made, for, as the experienced researcher realizes, issues of considerable moment, such as the theoretical alternatives in Cattell's and Eysenck's structuring of neuroticism and extraversion, hinge alternatively on rotational resolutions (p. 223).

The typical flow of decisions to be made in the use of factor analysis is:

- Technique of factoring
- Number of factors to be used
- Choice of rotation
- Interpretation and usage of information from the analysis.

Considerable considerable guidance is available for the choice of techniques and in deciding upon the number of factors to retain. However, the choice of which rotation technique to use has received less attention. Furthermore, most of the advice is technical in nature.

What we need is an understanding of which rotation is appropriate depending upon the specific use of the researcher. The purpose of this paper is to provide a framework of choosing a specific factor rotation based on a usage framework.

<u>Why A Particular Rotation?</u>

As Strang puts it, "for any orthogonal matrix of order p, the factor matrix $\tilde{F} = FQ$ accounts for exactly the same correlations $\tilde{F}\tilde{F}^T = FQQ^TF^T = FF^T$ that F does. Therefore, F and $\tilde{F}$ are completely interchangeable. In a typical problem, the original factors may have substantial loadings on dozens of variables, and such a factor is practically impossible to interpret. It has mathematical meaning as a vector of f_i, but no useful meaning to a social scientist. Therefore he tries to choose a rotation... ." (Strang, 1976, p. 216).

For marketing researchers, the hard work really begins. Interpretation is the key to meaningful use of factor analysis. And, a key to meaningful interpretation is the perspective or rotation brought to bear upon the factor solution.

In an applied field such as market research, interpretation of the data analysis is a key to the meaningful use of factor analysis. The hard work really begins after the factors have been extracted and the market researcher must decide on which rotation technique to use. Unfortunately, the lack of determinancy and choice of infinite analytic solutions creates a problem of ensuring the right choice which is based on nonstatistical criteria.

Let us consider an example to illustrate the importance played by rotation in interpretation. In an analysis of some census tract information (the variables being total population, median school years, total employment, miscellaneous professional services, and value of median value house) Harman (1967) obtained a two-factor solution. The factor loadings he obtained are shown in Exhibit 1. As shown in the inset to Exhibit 1, the two-factor solution explain 93.5 percent of the variance explained by the original five variables. But it is difficult to interpret the two-factor solution. All the variables, except variable 4, load moderately or very heavily on both the factors. If our interest is in drawing a rich meaning from the factors, we would seek to achieve Thurstone's (1947) simple structure to attain which rotation is needed. A simple rotation that is visually quite obvious, in this simple two factor, five variables example, is shown in Exhibit 2. Such a rotation leads to the factor loadings matrix shown in the inset to Exhibit 2. It is very clear both from the graph as well as from the loadings matrix that factor 1 (M1) is a clean composite of variables 2 and 5. Factor 2 (M2) is loaded upon heavily by variables 1 and 3. Variable 4 loads heavily on factor 1 and moderately on factor 2, leading to some possible ambiguity. Factor M1 can be interpreted (as in Harman) as a factor explaining aspects of overall size of the tract. Factor M2 appears to explain aspects of the quality (or attainment level) of a census tract. The value of factor rotation in correctly interpreting a factor analysis solution is thus immeasurable.

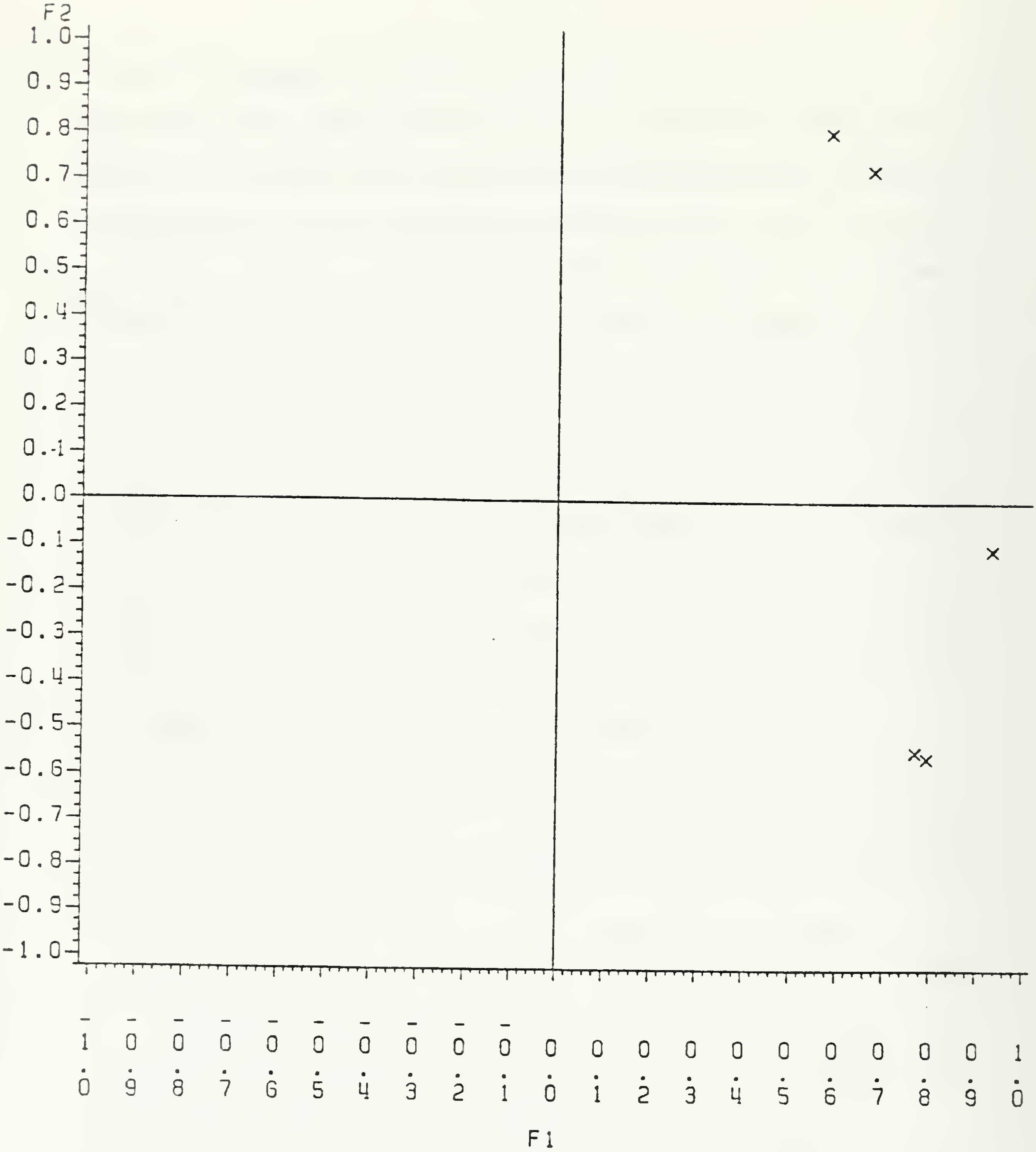
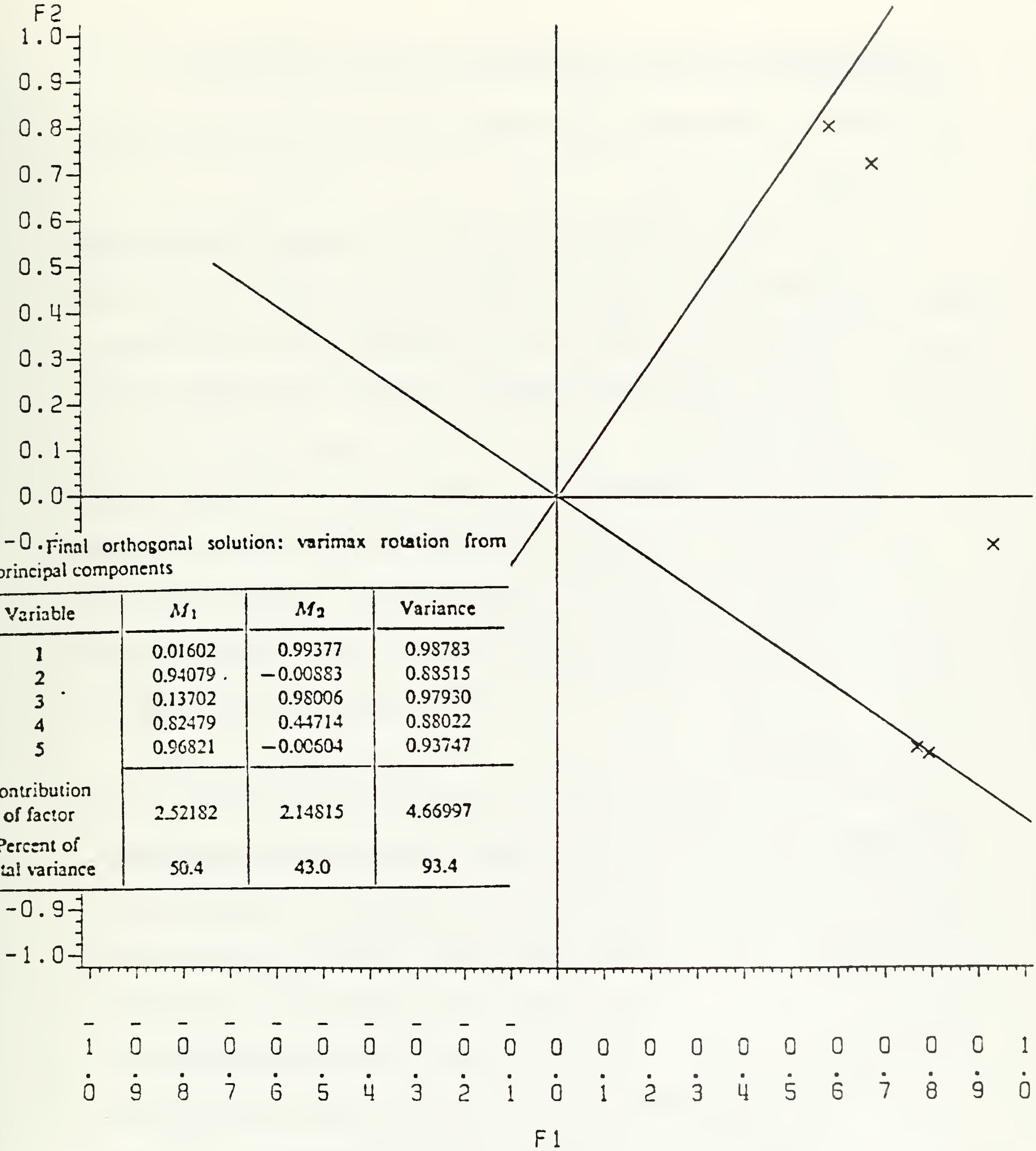


EXHIBIT 1: EXAMPLE WITHOUT ROTATION

Principal-component solution

Variable	F_1	F_2	Variance
1	0.58096	0.80642	0.98783
2	0.76705	−0.54478	0.88515
3	0.67243	0.72604	0.97930
4	0.93239	−0.10429	0.88023
5	0.79115	−0.55816	0.93746
Contribution of factor	2.87331	1.79666	4.66997
Percent of	[illegible]	[illegible]	[illegible]



Final orthogonal solution: varimax rotation from o principal components

Variable	M_1	M_2	Variance
1	0.01602	0.99377	0.98783
2	0.94079	−0.00883	0.88515
3	0.13702	0.98006	0.97930
4	0.82479	0.44714	0.88022
5	0.96821	−0.00604	0.93747
Contribution of factor	2.52182	2.14815	4.66997
Percent of total variance	50.4	43.0	93.4

EXHIBIT 2: WITH ROTATION

Uses for Factor Analysis in Marketing and their Relationship to Rotation

There are four major types of uses for factor analysis in marketing research. These are:

1. To provide insights into the more fundamental and underlying factors which create a structure of systematic relationships among observed phenomena. For example, ascertaining the factors underlying systematic preferences for various types of liquors.

2. To provide a typology or classification of a large number of entities. For example, development of a typology of magazine readers based on their reading of over 50 popular magazines.

3. To bring out in bold relief salient relationships so that researchers can cull the relevant variables before further analysis.

In the task of building better forecasting and predictive models, one seeks to build parsimonious models. Factor analysis provides a way of selecting the relevant predictor variables from a larger set of variables. This pruning is particularly critical when given only a few observations, we need to reduce the large number of predictor variables to an optimal number to preserve the degrees of freedom.

4. To transform data so as to make them more representative of the assumptions of powerful predictive models.

Example 1: To transform data to meet the conditions of normality, independence, and random error assumptions of regression analysis.

Example 2: To transform available data to meet the conditions of independence and homogeneity indiscriminant analysis.

Yet another possible use could be a factor analytic solution as part of a higher order factor analysis (Rummel, 1970). In higher order factor analysis, a first stage solution identifies a factor structure (reduced space) that explains the interrelationships in the original data (say among a large set of variables). The analyst expects or wishes to explore the possibility that an even more parsimonious description of the interrelationships is possible. Thus, he would perform a higher order factor analysis of a factor solution to obtain this more parsimonious, and higher order solution. We have summarized this discussion in Exhibit 3.

Several rotation schemes have been developed and incorporated in the numerous statistical analysis packages available to users. These schemes are broadly classified into (i) oblique, and (ii) orthogonal rotation schemes. Furthermore, one can rotate the factor structure to simplify and therefore get better understanding of the (a) variables or the (2) underlying factors.

The outcome of using an orthogonal rotation is the development of a reduced factor space, where the factors are orthogonal (or "independent"). Oblique rotations, in general, lead to factors that are not orthogonal (i.e., the axes of the new reduced space are correlated).

Further, each rotation scheme, as mentioned earlier, has a different objective that it seeks to satisfy optimally. We provide a

RELATIONSHIP BETWEEN FACTORS	FOCUS ON: VARIABLES	FOCUS ON: FACTORS
INDEPENDENT	SEARCH FOR RELEVANT VARIABLES (3)	DATA REDUCTION FOR FURTHER ANALYSIS (4)
CORRELATED	TYPOLOGY OR INSTERING OF VARIABLES (2)	UNDERLYING PREFERENCE DIMENSIONS HIGHER ORDER FACTOR ANALYSIS (1)

EXHIBIT 3: A TYPOLOGY OF MARKETING APPLICATIONS OF FACTOR ANALYSIS

brief description of the criterion for each scheme in the Appendix. These criteria lead to either an ease of interpretation by focusing on the emergent structural relationships between different variables, or by focusing on the interpretation of the factors that underlie the observed (or manifest) variables.

Based on the criterion that is used in each of the rotation schemes, we have classified the more generally available and popular schemes on the basis of (i) whether they perform oblique or orthogonal rotations, and (ii) whether they aid interpretation of the variables or the underlying factors. Exhibit 4 provides this classification.

As can readily be seen there is a match between Exhibits 3 and 4. Thus, we have identified a typology of possible marketing applications of factor analysis that will allow a choice of the appropriate rotation. To make our exposition clearer, and the framework easy to use, let us consider some marketing examples.

Stoetzel in 1960, was interested in identifying the dimensions underlying preferences for liquor among French consumers. His concern was with the factors (or the latent dimensions) based on which preferences were developed for different liquors. His concern was not with specific liquors but with the underlying preference dimensions on which all types of liquors are preferred. He was thus interested in identifying the latent factors and the relationships among these factors. This clearly is an example of box 1 in Exhibit 4. A correct choice should have been the use of an obliqued rotation which focuses on simplification or interpretation of factors. This includes oblique Varimax or its variations. However, Stoetzel used an orthogonal

ROTATION TYPES	FOCUS ON: VARIABLES	FOCUS ON: FACTORS
ORTHOGONAL	QUARTIMAX (3)	VARIMAX (4)
OBLIQUE	OBLIMAX (2)	PROMAX OBLIMIN COVARIMIN BINORMAMIN OBLIQUE VARIMAX (Harris Kaiser Case II) QUARTIMIN BIQUARTIMIN (1)

EXHIBIT 4: A CLASSIFICATION OF ROTATION SCHEMES

rotation. Having developed a three factor solution, he labeled these factors as (1) sweetness strength, (2) low price-high price, and (3) regional popularity. There could be an interaction between factors 1 and 3, and 2 and 3. An oblique rotation would bring this out in a clear fashion. An orthogonal rotation on the other hand would force factor independence and obscure any such relationships.

Wells and Sheth in 1974, reported on a study typical of studies used for clustering and segmentation. Their primary objective was to analyze the frequency of readership of 30 magazines and identify a classification of magazine types. This was to further enable them to draw implications regarding the commonality of audience provided by each grouping of magazines. Based on their analysis they report groupings of magazines. Some of these are (i) Car and Driver, Road and Track, Motor Trend and Hot Rod, (ii) Fortune, Forbes, Time, and Business Week, (iii) Field and Stream, Outdoor Life, and Sports Afield, and so on. Their focus is on the variables (the magazines for which they obtained readership frequency data. By allowing an oblique rotation, they could have permitted a more natural interpretation of their analysis. By performing a Varimax (orthogonal) rotation, they precluded the possibility of magazines appearing in more than one group. For example, Reader's Digest does not have high loadings on any of the factors, in spite of attempting to force it to load heavily on just one factor (using Varimax). Its positive loadings are divided among a "news group" (U.S. News and Newsweek), a "general reading" group (Life, Look, Saturday Evening Post), and a "men's fiction" group (Argosy and True). But, these loadings are

extremely likely to change, if our objective allows the spreading of loadings by permitting correlated (or oblique) factors.

For cell 3, the study by Twedt (1952) of advertising readership is a good example. He was interested in identifying the determinants of advertising readership. His was a search for the most promising variables that correlated with advertising readership. He obtained a recall-based readership measure on 122 advertisements of 1/4 page or more in size from a single issue of American Builder, a trade magazine. Based on judgment, 34 variables describing the mechanical and content aspects of these advertisements were chosen. By studying the correlations between these 34 variables and his readership variable, he chose 19 predictor variables (out of 34). A factor analysis of these 19 variables along with the criterion variable yielded 6 meaningful factors. These factors were orthogonally rotated before interpretation. He wished to identify the variables that were the most relevant as predictor variables, and could be used as predictor variables in a multiple regression model. So his focus was on the variable and he wished to obtain independent (or orthogonal) variables to use in his multiple regression model. This model resulted in a multiple of 0.75. When an additional six variables were added to the model, the multiple R increased only by 0.04 which suggests the adequacy of his factor analysis solution in identifying the relevant variables which were determinant of advertising readership.

Deshpande (1982), provides us an example for cell 4. His study was aimed at understanding the organizational context of market research use. He wished to develop a multiple regression model that

would predict research utilization. Factor analysis was used to develop a set of predictor variables for this model. These predictor variables were thus to be uncorrelated from a set of 23 variables (that were concerned with measuring different elements of perceptions of organizational structure) he developed a smaller set of indices that would be the latent variables giving rise to the manifest variables. These indices (or factor scores) were providing the meaning of being the latent, or causal, dimensions of organizational structure perceptions. His focus was clearly on identifying and modeling the factor structure. His factors (or indices) were to be orthogonal, and thus clearly the method of rotation (Varimax) chosen by him follows from our framework as the appropriate choice.

Conclusions

Hair, Anderson, Tattan, and Grablowsky (1979, p. 230), note "No specific rules have been developed to guide the analyst in selecting a particular ... rotational technique." We urge our readers not to choose a rotation scheme just because Varimax is a familiar and common scheme, but to reason through the raison d'êtere of their analysis, to arrive at the appropriate rotation scheme which will aid in better (and more correct) substantive interpretation. Towards this end, in this paper, we have provided a usage based choice framework explicitly for this purpose.

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APPENDIX

A Technical Summary of Major Rotation Schemes

ORTHOGONAL SCHEMES

Scheme	Objective Function	Comments
QUARTIMAX	$\text{Max } Q = \sum_{j=1}^{m} \sum_{\ell=1}^{p} \alpha_{j\ell}^{4}$ where, $\alpha_{j\ell}$ = factor loading of variable x_j on factors S_ℓ, m = number of variables, p = number of factors.	This function increases the high, and decreases the middle loadings for a variable. It approximates simple structure, but has a bias toward loading more variance on the first factor.
VARIMAX	$\text{Max } V = m \sum_{\ell=1}^{p} \sum_{j=1}^{m} \left(\frac{\alpha_{j\ell}}{h_j}\right)^4 - \sum_{\ell=1}^{p} \left(\sum_{j=1}^{m} \frac{\alpha_{j\ell}^2}{h_j^2}\right)^2$ where, v = variance of normalized factors, h_j^2 = communality of variable x_j.	Very commonly used (sometimes inappropriately) orthogonal rotation. Approximates simple structure.

OBLIQUE SCHEMES

OBLIMAX	Max K = kurtosis $= \dfrac{\sum_{j=1}^{m} \sum_{\ell=1}^{p} \alpha_{j\ell}^{4}}{\left(\sum_{j=1}^{m} \sum_{\ell=1}^{p} \alpha_{j\ell}\right)^4}$ where $\alpha_{j\ell}$ = oblique reference structure loading.	Attempts to increase the number of high and low loadings and decrease those in the middle. Good to use only if data have clean simple structure.

OBLIQUE SCHEMES

Scheme	Objective Function	Comments
QUARTIMIN	$\text{Min } Q = \sum_{1<a=1}^{p} \sum_{j=1}^{m} \alpha_{j\ell}^2 \alpha_{jq}^2$ where $\alpha_{j\ell}$ = oblique reference structure loading.	Attempts to increase the high loadings of a variable on one factor and decrease its loadings on others. Results oftentimes in highly intercorrelated (0.5, often >0.7) factors.
CCOVARIMIN	$\text{Min } C = \sum_{1<q=1}^{p} \{m \sum_{j=1}^{m} (\frac{\alpha_{j\ell}^2}{h_j^2})(\frac{\alpha_{jq}^2}{h_j^2})$ $- [\sum_{j=1}^{m} (\frac{\alpha_{j\ell}^2}{h_j^2})][\sum_{j=1}^{m} (\frac{\alpha_{jq}^2}{h_j^2})]$	Tends to produce solutions very close to orthogonal Varimax solutions (i.e., with low intercorrelations).
BIQUARTIMIN	$\text{Min } B = Q + \frac{C}{m}$ Q = Quartimin function C = Covarimin function	Good blend between Quartimin and Covarimin. Generally provides a more satisfactory simple structure solution than either of those (in terms of interfactor correlations and factor loadings).

OBLIQUE SCHEMES

Scheme	Objective Function	Comments
OBLIMIN	$\text{Min } B^* = \beta_1 Q + \beta_2 \frac{C}{m}$ or $\text{Min } B^* = \sum_{\ell<q=1}^{p} \{m \sum_{j=1}^{m} (\frac{\alpha_{j\ell}^2}{h_j^2}) - \gamma$ $\cdot [\sum_{j=1}^{m} (\frac{\alpha_{j\ell}^2}{h_j^2})] [\sum_{j=1}^{m} (\frac{\alpha_{jq}^2}{h_j^2})]\}$ $\gamma = \frac{\beta_2}{\beta_1+\beta_2}$	This is a general class of Biquartimin functions. With $\gamma = 0.5$ you get Biquartimin solutions, with $\gamma = 1.0$, you get Covarimin and if $\gamma = 0$, you get Quartimin.
BINORMAMIN	$\text{Min } B^{**} = \sum_{\ell<q=1}^{p}$ $\cdot \{\frac{\sum_{j=1}^{m} (\alpha_{j\ell}^2/h_j^2)(\alpha_{jq}^2/h_j^2)}{[\sum_{j=1}^{m} (\alpha_{j\ell}^2/h_j^2)][\sum_{j=1}^{m} (\alpha_{jq}^2/h_j^2)]}\}$	This appears best suited for data with either very clear, or very complex structure. For in-between data, use Biquartimin.

OBLIQUE SCHEMES

Scheme	Objective Function	Comments
Oblique Varimax (or Harris-Kaiser Case II)	$P = QM^{1/2}T_1D_1$ where P is the ultimate factor pattern matrix, Q is an orthonormal matrix of latent vectors, M is a diagonal matrix of latent roots, T_1 is a square and orthonormal transformation matrix based on criterion such as Varimax, Equimax, or Quartimax, and $D_1^2 = P'P$	Commonly used oblique scheme. Generally produces satisfactory simple structure solutions.
MAXPLANE	Max N where N is the number of variables falling within a prespecified hyperplane width.	It is more efficient on large samples and yields solutions close to graphical intuition.
PROMAX	First obtains a Varimax solution, then relaxes orthogonality to obtain better fit to simple structure.	If data dictates orthogonal solution, Promax usually reaches this solution. Its solutions approach simple structure.